Theorem: It is not the case that there are natural numbers n and m such that 3m + 5n = 12

Proof: We will prove the theorem by contradiction. Assume it is the case that $(\exists m \in N)(\exists n \in N)(3m + 5n = 12)$

We know 12 = (3)(4) so by the definition of divisibility 3 | 12. The division theorem states that $a \mid b$ iff b = am + r for some natural number m where r = 0.

Substituting our values into the division theorem, 12 = 3m + 5n where r = 5n. We know 3 | 12 so 5n must be 0, however n cannot be 0 since n is defined as a natural number.

This is a contradiction so our assumption is false which means its negation is true. So it is not the case that $(\exists m \in N)(\exists n \in N)(3m + 5n = 12)$. The theorem is proved.